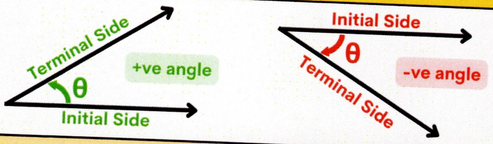
TRIGNOMETRIC FUNCTIONS

Angles



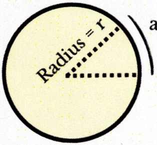
Units of Angle (System) measurement

- For Small Angles : **Degree** (Sexagesimal) **or Radian** (Circular)
- For Large Angles : **Revolution**

Angle is repeated after every $2n\pi$

Degree	30	45	60	90	180	270	360
Radian	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π

$$\text{Radian measure} = \frac{\pi}{180} \times \text{Degree measure}$$



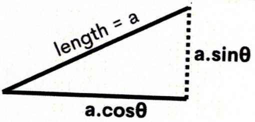
arc length = l

$$\theta = \frac{l}{r}$$

REMEMBER!

1 radian ≈ 57 degrees
1.57 radian ≈ 90°
3.14 radian ≈ 180°

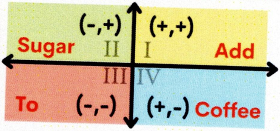
$\cos\theta$ & $\sin\theta$



$\sin x = 0 \Rightarrow x = n\pi$

$\cos x = 0 \Rightarrow x = (2n+1)\pi/2$

$-1 \leq \cos\theta,\ \sin\theta \leq +1$



Signs in different quadrants

(cos θ, sin θ)

Add Sugar To Coffee : **All** Positive, **Sin** Positive, **Tan** Positive, **Cos** Positive

f(x)	sin	cos	tan	cosec	sec	cot
0°	0	1	0	N.D.	1	N.D.
30°	1/2	√3/2	1/√3	2	2/√3	√3
45°	1/√2	1/√2	1	√2	√2	1
60°	√3/2	1/2	√3	2/√3	2	1/√3
90°	1	0	N.D.	1	N.D.	0
120°	√3/2	-1/2	-√3	2/√3	-2	-1/√3
180°	0	-1	0	N.D.	-1	N.D.
270°	-1	0	N.D.	-1	N.D.	0
360°	0	1	0	N.D.	1	N.D.

Allied Angles

- θ & $n\pi \pm \theta$ are allied angles.
- Similarly θ & $(2n + 1)\frac{\pi}{2} \pm \theta$

DON'T FORGET

Generalised Angle placement

• $n\pi - \theta$, n = odd • $(4n + 1)\frac{\pi}{2} + \theta$, n = whole **QUAD. II**	• $n\pi + \theta$, n = even • $(4n + 1)\frac{\pi}{2} - \theta$, n = whole **QUAD. I**
• $n\pi + \theta$, n = odd • $(4n + 3)\frac{\pi}{2} - \theta$, n=whole **QUAD. III**	• $n\pi - \theta$, n = even • $(4n + 3)\frac{\pi}{2} + \theta$, n=whole **QUAD. IV**

e.g. $\sin(5\pi - \theta) = \sin\theta$ **[Quad II]**

- **Remember, $\sin(-\theta) = -\sin\theta$; $\cos(-\theta) = \cos\theta$**

Complimentary Angles

- For multiples of $\pi/2$, we use complimentary angles
- sin ⇔ cos ; cosec ⇔ sec ; tan ⇔ cot
- Sign of resultant function according to L.H.S. function

$\cos\left(\frac{5\pi}{2} - \theta\right) = \sin\theta$ lies in first quadrant and sign of cos = +ve	$\tan\left(\frac{3\pi}{2} + \theta\right) = -\cot$ lies in fourth quadrant and sign of cos = +ve whereas sin = -ve

All Trigonometric Formulas

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$\cot^2\theta + 1 = \text{cosec}^2\theta$$

DON'T FORGET!

Double Angle Formulas

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 1 - 2\sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Triple angle formulas

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$
$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$
$$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

Power reducing formulas

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$
$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$
$$\tan^2\theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

Half Angle Formula

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{2}}$$
$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1 + \cos\theta}{2}}$$
$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}$$
$$= \frac{1 - \cos\theta}{\sin\theta}$$
$$= \frac{\sin\theta}{1 + \cos\theta}$$

All Trigonometric Formulas

Angle Addition Formulas

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\tan\left(\frac{\pi}{4} + \theta\right) = \frac{1 + \tan\theta}{1 - \tan\theta}$$

$$\tan\left(\frac{\pi}{4} - \theta\right) = \frac{1 - \tan\theta}{1 + \tan\theta}$$

Product to Sum formulas

$$\sin A.\sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$$

$$\cos A.\cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

$$\sin A.\cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$$

$$\cos A.\sin B = \frac{1}{2}[\sin(A + B) - \sin(A - B)]$$

All Trigonometric Formulas

Sum to Product Formulas

$$\sin A + \sin B = 2\left[\sin\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right)\right]$$

$$\sin A - \sin B = 2\left[\sin\left(\frac{A-B}{2}\right).\cos\left(\frac{A+B}{2}\right)\right]$$

$$\cos A + \cos B = 2\left[\cos\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right)\right]$$

$$\cos A - \cos B = -2\left[\sin\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right)\right]$$

Conditional identity, If A+B+C = π

$$\tan A + \tan B + \tan C = \tan A.\tan B.\tan C$$

$$\cot B.\cot C + \cot C.\cot A + \cot A.\cot B = 1$$

$$\tan\frac{B}{2}.\tan\frac{C}{2} + \tan\frac{C}{2}.\tan\frac{A}{2} + \tan\frac{A}{2}.\tan\frac{B}{2} = 1$$

$$\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}.\cot\frac{B}{2}.\cot\frac{C}{2}$$